

Mathematics, the Common Core, and Language: Recommendations for Mathematics Instruction for ELs Aligned with the Common Core

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1. Introduction

This paper outlines recommendations for meeting the challenges in developing mathematics instruction for English Learners (ELs) that is aligned with the Common Core Standards. The recommendations are motivated by a commitment to improving mathematics learning *through language* for all students and especially for students who are learning English. These recommendations are not intended as recipes or quick fixes, but rather as principles to help to guide teachers, curriculum developers, and teacher educators in developing their own approaches to supporting mathematical reasoning and sense making for students who are learning English.

These recommendations for teaching practices are based on research that often runs counter to commonsense notions of language. The first issue is the term *language*. There are multiple uses of the term *language*: to refer to the language used in classrooms, in the home and community, by mathematicians, in textbooks, and in test items. It is crucial to clarify how we use the term, what set of phenomena we are referring to, and which aspects of these phenomena we are focusing on. Many commentaries on the role of academic language in mathematics teaching practice reduce the meaning of the term to single words and the proper use of grammar (for example, see Cavanagh, 2005). In contrast, work on the language of specific disciplines provides a more complex view of mathematical language (e.g., Pimm, 1987) as not only specialized vocabulary (new words and new meanings for familiar words) but also as extended discourse that includes syntax and organization (Crowhurst, 1994), the mathematics register (Halliday, 1978), and discourse practices (Moschkovich, 2007c). Theoretical positions in the research literature in mathematics education range from asserting that mathematics is a universal language, to claiming that mathematics is itself a language, to describing how mathematical language is a problem. Rather than joining in these arguments, I use a sociolinguistic framework to frame this essay. From this theoretical perspective, language is a socio-cultural-historical activity, not a thing that can either be mathematical or not, universal or not. I use the phrase “the language of mathematics” not to mean a list of vocabulary or technical words with precise meanings but the communicative competence necessary and sufficient for competent participation in mathematical discourse practicesⁱ.

It is difficult to make generalizations about the instructional needs of all students who are learning English. Specific information about students’ previous instructional experiences in mathematics is crucial for understanding how bilingual learners communicate in mathematics

classrooms. Classroom instruction should be informed by knowledge of students' experiences with mathematics instruction, their language history, and their educational background. In addition to knowing the details of students' experiences, research suggests that high-quality instruction for ELs that supports student achievement has two general characteristics: a view of language as a resource, rather than a deficiency; and an emphasis on academic achievement, not only on learning English (Gándara and Contreras, 2009).

Research provides general guidelines for instruction for this student population. Since students who are labeled as ELs, who are learning English, or who are bilingual are from non-dominant communities, they need access to curricula, instruction, and teachers proven to be effective in supporting academic success for this student population. The general characteristics of such environments are that curricula provide “abundant and diverse opportunities for speaking, listening, reading, and writing” and that instruction “encourage students to take risks, construct meaning, and seek reinterpretations of knowledge within compatible social contexts” (Garcia & Gonzalez, 1995, p. 424). Teachers with documented success with students from non-dominant communities share some characteristics: a) a high commitment to students' academic success and to student-home communication, b) high expectations for all students, c) the autonomy to change curriculum and instruction to meet the specific needs of students, and d) a rejection of models of their students as intellectually disadvantagedⁱⁱ.

Research on language that is specific to mathematics instruction for this student population provides several guidelines for instructional practices for teaching ELs mathematics. Mathematics instruction for ELs should: 1) treat language as a resource, not a deficit (Gándara and Contreras, 2009; Moschkovich, 2000); 2) address much more than vocabulary and support ELs' participation in mathematical discussions as they learn English (Moschkovich, 1999, 2002, 2007a, 2007b, 2007d); and 3) draw on multiple resources available in classrooms – such as objects, drawings, graphs, and gestures – as well as home languages and experiences outside of school. This research shows that ELs, even as they are learning English, can participate in discussions where they grapple with important mathematical contentⁱⁱⁱ. Instruction for this population should not emphasize low-level language skills over opportunities to actively communicate about mathematical ideas. One of the goals of mathematics instruction for ELs should be to support all students, regardless of their proficiency in English, in participating in discussions that focus on important mathematical concepts and reasoning, rather than on pronunciation, vocabulary, or low-level linguistic skills. By learning to recognize how ELs express their mathematical ideas as they are learning English, teachers can maintain a focus on mathematical reasoning as well as on language development.

Research also describes how mathematical communication is more than vocabulary. While vocabulary is necessary, it is not sufficient. Learning to communicate mathematically is not merely or primarily a matter of learning vocabulary. During discussions in mathematics classrooms, students are also learning to describe patterns, make generalizations, and use representations to support their claims. The question is not whether students who are ELs should learn vocabulary but rather how instruction can best support students as they learn both vocabulary and mathematics. Vocabulary drill and practice is not the most effective instructional

practice for learning either vocabulary or mathematics. Instead, vocabulary and second-language-acquisition experts describe vocabulary acquisition in a first or second language as occurring most successfully in instructional contexts that are language-rich, actively involve students in using language, require both receptive and expressive understanding, and require students to use words in multiple ways over extended periods of time (Blachowicz and Fisher, 2000; Pressley, 2000). In order to develop written and oral communication skills students need to participate in negotiating meaning (Savignon, 1991) and in tasks that require output from students (Swain, 2001). In sum, instruction should provide opportunities for students to actively use mathematical language to communicate about and negotiate meaning for mathematical situations.

The recommendations provided in this paper focus on teaching practices that are simultaneously: a) aligned with the Common Core Standards for mathematics, b) support students in learning English, and c) support students in learning important mathematical content. Overall, the recommendations address the following questions: How can instruction provide opportunities for mathematical reasoning and sense making for students who are learning English? What instructional strategies support ELs' mathematical reasoning and sense making skills? How can instruction help EL students communicate their reasoning effectively in multiple ways?

2. Alignment with Common Core State Standards

The Common Core State Standards (CC) provide guidelines for how to teach mathematics for understanding by focusing on students' mathematical reasoning and sense making. Here I will only summarize four emphases provided by the CC to describe how mathematics instruction for ELs needs to begin by following CC guidelines and taking these four areas of emphasis seriously.

Emphasis #1 Balancing conceptual understanding and procedural fluency

Instruction should a) balance student activities that address both important conceptual and procedural knowledge related to a mathematical topic and b) connect the two types of knowledge.

Emphasis #2 Maintaining high cognitive demand

Instruction should a) use high-cognitive-demand math tasks and b) maintain the rigor of mathematical tasks throughout lessons and units.

Emphasis #3 Developing beliefs

Instruction should support students in developing beliefs that mathematics is sensible, worthwhile, and doable.

Emphasis #4 Engaging students in mathematical practices

Instruction should provide opportunities for students to engage in eight different mathematical practices: 1) Make sense of problems and persevere in solving them, 2) reason abstractly and quantitatively, 3) construct viable arguments and critique the reasoning of others, 4) model with

mathematics, 5) use appropriate tools strategically, 6) attend to precision, 7) look for and make use of structure, and 8) look for and express regularity in repeated reasoning.

We can see from these areas of emphasis that students should be focusing on making connections, understanding multiple representations of mathematical concepts, communicating their thought processes, and justifying their reasoning. Several of the mathematical practices involve language and discourse (in the sense of talking, listening, reading, and writing), in particular practices #3 and #8. In order to engage students in these mathematical practices, instruction needs to include time and support for mathematical discussions and use a variety of participation structures (teacher-led, small group, pairs, student presentations, etc.) that support students in learning to participate in such discussions.

According to a review of the research (Hiebert & Grouws, 2007), mathematics teaching that makes a difference in student achievement and promotes conceptual development in mathematics has two central features: one is that teachers and students attend explicitly to concepts, and the other is that teachers give students the time to wrestle with important mathematics. Mathematics instruction for ELs should follow these general recommendations for high-quality mathematics instruction to focus on mathematical concepts and the connections among those concepts and to use and maintain high-cognitive-demand mathematical tasks, for example, by encouraging students to explain their problem-solving and reasoning (AERA, 2006; Stein, Grover, and Henningsen, 1996).

One word of caution: concepts can often be interpreted to mean definitions. However, paying explicit attention to concepts does not mean that teachers should focus on providing definitions or stating general principles. Instead the CC and the National Council of Teachers of Mathematics (NCTM) Standards provide multiple examples of how instruction can focus on important mathematical concepts (e.g. equivalent fractions or the meaning of fraction multiplication, etc.). Similarly, the CC and NCTM also provide examples of how students can show their understanding of concepts (conceptual understanding) not by giving a definition or describing a procedure, but by using multiple representations. For example, students can show conceptual understanding by using a picture of a rectangle as an area model to *show* that two fractions are equivalent or how multiplication by a positive fraction smaller than one makes the result smaller, and pictures can be accompanied by oral or written explanations.

The preceding examples point to several challenges that students face in mathematics classrooms focused on conceptual understanding. Since conceptual understanding is most often made visible by showing a solution, describing reasoning, or explaining “why,” instead of simply providing an answer, the CC shifts expectation for students from carrying out procedures to communicating their reasoning. Students are expected to a) communicate their reasoning through multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.), b) engage in productive pictorial, symbolic, oral, and written group work with peers, c) engage in effective pictorial, symbolic, oral, and written interactions with teachers, d) explain and demonstrate their knowledge using emerging language, and e) extract meaning from written mathematical texts. The main challenges for teachers teaching mathematics are to teach

for understanding, support students to use multiple representations, and support students in using emerging and imperfect language to communicate about mathematical concepts. Since the CC documents already provide descriptions of how to teach mathematics for understanding and use multiple representations, the recommendations outlined below will focus on how to connect mathematical content to language, in particular through “engaging students in mathematical practices” (Emphasis #4).

3. Recommendations for Connecting Mathematical Content to Language

Recommendation #1: Focus on students’ mathematical reasoning, not accuracy in using language.

Instruction should focus on uncovering, hearing, and supporting students’ mathematical reasoning, not on accuracy in using language (either English or a student’s first language). When the goal is to engage students in mathematical practices, student contributions are likely to first appear in imperfect language. Teachers should not be sidetracked by expressions of mathematical ideas or practices expressed in imperfect language. Instead, teachers should first focus on promoting and privileging meaning, no matter the type of language students may use. Eventually, after students have had ample time to engage in mathematical practices both orally and in writing, instruction can then carefully consider how to move students toward accuracy.

As a teacher, it can be difficult to understand the mathematical ideas in students’ talk in the moment. However, it is possible to take time after a discussion to reflect on the mathematical content of student contributions and design subsequent lessons to address these mathematical concepts. But, it is only possible to uncover the mathematical ideas in what students say if students have the opportunity to participate in a discussion and if this discussion is focused on mathematics. Understanding and re-phrasing student contributions can be a challenge, perhaps especially when working with students who are learning English. It may not be easy (or even possible) to sort out what aspects of what a student says are due to the student’s conceptual understanding or the student’s English language proficiency. However, if the goal is to support student participation in a mathematical discussion and in mathematical practices, determining the origin of an error is not as important as listening to the students and uncovering the mathematical content in what they are saying.

Recommendation #2: Shift to a focus on mathematical discourse practices, move away from simplified views of language.

In keeping with the CC focus on *mathematical practices* (Emphasis #4) and research in mathematics education, the focus of classroom activity should be on student participation in mathematical discourse practices (explaining, conjecturing, justifying, etc.). Instruction should move away from simplified views of language as words, phrases, vocabulary, or a list of definitions. In particular, teaching practices need to move away from oversimplified views of language as vocabulary and leave behind an overemphasis on correct vocabulary and formal language, which limits the linguistic resources teachers and students can use in the classroom to learn mathematics with understanding. Work on the language of disciplines provides a

complex view of mathematical language as not only specialized vocabulary – new words and new meanings for familiar words – but also as extended discourse that includes syntax, organization, the mathematics register, and discourse practices. Instruction needs to move beyond interpretations of the mathematics register as merely a set of words and phrases that are particular to mathematics. The mathematics register includes styles of meaning, modes of argument, and mathematical practices and has several dimensions such as the concepts involved, how mathematical discourse positions students, and how mathematics texts are organized.

Another simplified view of language is the belief that precision lies primarily in individual word meaning. For example, we could imagine that *attending to precision* (mathematical practice #6) means using two different words for the set of symbols “ $x+3$ ” and the set of symbols “ $x+3 = 10$.” If we are being precise at the level of individual word meaning, the first is an “expression” while the second is an “equation.” However, attending to precision is not so much about using the perfect word; a more significant mathematical practice is making claims *about precise situations*. We can contrast the claim “Multiplication makes bigger,” which is not precise, with the question and claim “When does multiplication make the result bigger? Multiplication makes the result bigger when you multiply by a number greater than 1.” Notice that when contrasting these two claims, precision does not lie in the individual words nor are the words used in the more precise claim fancy math words. Rather, the precision lies in the *mathematical practice* of specifying when the claim is true. In sum, instruction should move away from interpreting precision to mean using the precise word, and instead focus on how precision works in mathematical practices.

One of the eight mathematical practices, “Attend to precision” (Number 6), is open to such multiple interpretations of the term “precision.” It is important to consider what we mean by *precision* for all students learning mathematics, since all students are likely to need time and support for moving from expressing their reasoning and arguments in imperfect form. However, it is essential for teachers of ELs to consider when and how to focus on precision for ELs. Although students’ use of imperfect language is likely to interact with teachers’ own multiple interpretations of precision, we should not confuse the two. In particular, we should remember that precise claims can be expressed in imperfect language and that attending to precision at the individual word meaning level will get in the way of students’ expressing their emerging mathematical ideas. More work is needed to clarify how to guide practitioners in helping students become more precise in their language over time.

Recommendation #3: Recognize and support students to engage with the complexity of language in math classrooms.

Language in mathematics classrooms is complex and involves a) multiple modes (oral, written, receptive, expressive, etc.), b) multiple representations (including objects, pictures, words, symbols, tables, graphs, etc.), c) different types of written texts (textbooks, word problems, student explanations, teacher explanations, etc.), d) different types of talk (exploratory and expository), and e) different audiences (presentations to the teacher, to peers, by the teacher, by peers, etc.). “Language” needs to expand beyond talk to consider the interaction of the three

semiotic systems involved in mathematical discourse – natural language, mathematics symbol systems, and visual displays. Instruction should recognize and strategically support EL students' opportunity to engage with this linguistic complexity.

Instruction needs to distinguish among multiple modalities (written and oral) as well as between receptive and productive skills. Other important distinctions are between listening and oral comprehension, comprehending and producing oral contributions, and comprehending and producing written text. There are also distinctions among different mathematical domains, genres of mathematical texts (for example word problems and textbooks). Instruction should support movement between and among different types of texts, spoken and written, such as homework, blackboard diagrams, textbooks, interactions between teacher and students, and interactions among students^{iv}. Instruction should: a) recognize the multimodal and multi-semiotic nature of mathematical communication, b) move from viewing language as autonomous and instead recognize language as a complex meaning-making system, and c) embrace the nature of mathematical activity as multimodal and multi-semiotic (Gutierrez et al., 2010; O'Halloran, 2005; Schleppegrell, 2010).

Recommendation #4: Treat everyday language and experiences as resources, not as obstacles.

Everyday language and experiences are not necessarily obstacles to developing academic ways of communicating in mathematics. It is not useful to dichotomize everyday and academic language. Instead, instruction needs to consider how to support students in connecting the two ways of communicating, building on everyday communication, and contrasting the two when necessary. In looking for mathematical practices, we need to consider the spectrum of mathematical activity as a continuum rather than reifying the separation between practices in out-of-school settings and the practices in school. Rather than debating whether an utterance, lesson, or discussion is or is not mathematical discourse, teachers should instead explore what practices, inscriptions, and talk mean to the participants and how they use these to accomplish their goals. Instruction needs to a) shift from monolithic views of mathematical discourse and dichotomized views of discourse practices and b) consider everyday and scientific discourses as interdependent, dialectical, and related rather than assume they are mutually exclusive.

The ambiguity and multiplicity of meanings in everyday language should be recognized and treated not as a failure to be mathematically precise but as fundamental to making sense of mathematical meanings and to learning mathematics with understanding. Mathematical language may not be as precise as mathematicians or mathematics instructors imagine it to be. Although many of us may be deeply attached to the precision we imagine mathematics provides, ambiguity and vagueness have been reported as common in mathematical conversations and have been documented as resources in teaching and learning mathematics (e.g., Barwell, 2005; Barwell, Leung, Morgan, & Street, 2005; O'Halloran, 2000; Rowland, 1999). Even definitions are not a monolithic mathematical practice, since they are presented differently in lower-level textbooks – as static and absolute facts to be accepted – while in journal articles they are presented as dynamic, evolving, and open to revisions by the mathematician. Neither should textbooks be seen as homogeneous. Higher-level textbooks are

more like journal articles in allowing for more uncertainty and evolving meaning than lower-level textbooks (Morgan, 2004), evidence that there are multiple approaches to the issue of precision, even in mathematical texts.

Recommendation #5: Uncover the mathematics in what students say and do.

Teachers need to learn how to recognize the emerging mathematical reasoning learners construct in, through, and with emerging language. In order to focus on the mathematical meanings learners construct rather than the mistakes they make or the obstacles they face, curriculum materials and professional development will need to support teachers in learning to recognize the emerging mathematical reasoning that learners are constructing in, through, and with emerging language (and as they learn to use multiple representations). Materials and professional development should support teachers so that they are better prepared to deal with the tensions around language and mathematical content, in particular a) how to uncover the mathematics in student contributions, b) when to move from everyday to more mathematical ways of communicating, and c) when and how to approach and develop “mathematical precision.” Mathematical precision seems particularly important to consider because it is one of the mathematical practices in the Common Core that can be interpreted in multiple ways (see Recommendations #2 and #4 for examples).

In sum, materials and professional development should raise teachers’ awareness about language, provide teachers with ways to talk explicitly about language, and model ways to respond to students. Teachers need support in developing the following competencies (Schleppegrell, 2010): using talk to effectively build on students’ everyday language as well as developing their academic mathematical language; providing interaction, scaffolding, and other supports for learning academic mathematical language; making judgments about defining terms and allowing students to use informal language in mathematics classrooms, and deciding when imprecise or ambiguous language might be pedagogically preferable and when not.

4. Closing Comments

Three issues are not addressed in the preceding recommendations: assessment, reading, and effective vocabulary instruction. Assessment is crucial to consider for ELs, because there is a history of inadequate assessment of this student population. LaCelle-Peterson and Rivera (1994, 2) write that ELs “historically have suffered from disproportionate assignment to lower curriculum tracks on the basis of inappropriate assessment and as a result, from over referral to special education (Cummins 1984; Durán 1989).” Previous work in assessment has described practices that can improve the accuracy of assessment in mathematics classrooms for this population. Assessment activities in mathematics should match the language of assessment with language of instruction, and include measures of content knowledge assessed through the medium of the language or languages in which the material was taught (LaCelle-Peterson and Rivera, 1994). Assessments should be flexible in terms of modes (oral and written) and length of time for completing tasks. Assessments should track content learning through oral reports and other presentations rather than relying only on written or one-time assessments. When students are first learning a second language, they are able to display content knowledge more easily by showing and telling, rather than through reading text or choosing from verbal options

on a multiple-choice test. Therefore, discussions with a student or observations of hands-on work will provide more accurate assessment data than written assessments. Evaluation should be clear as to the degree to which “fluency of expression, as distinct from substantive content” is being evaluated. This last recommendation raises an important challenge for assessing ELs’ mathematical proficiency: Classroom assessments based on mathematical discussions need to evaluate *content* knowledge as distinct from fluency of expression in English^V.

Learning to read mathematical texts is a topic that needs further research. Studies need to examine how ELs learn to read different mathematical texts (textbooks, word problems, etc.). In designing this research it is important to *differentiate between reading textbooks and reading word problems, two different genres in mathematical written discourse*. When working with children learning to read in English, it will be important to distinguish between children who are competent readers in a first language and children who are not. Lastly, since “language” seems to be so closely associated with “vocabulary,” we should develop principled and research-based best practices for supporting students in learning to use vocabulary in mathematics classrooms. Research should explicitly consider more and less successful ways for ELs to learn vocabulary in mathematics. This work will need to start by establishing what vocabulary assessment instruments are relevant to ELs learning mathematics.

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ⁱ I sometimes use the term “language(s)” as a reminder that there is no pure unadulterated language and that all language is hybrid.

ⁱⁱ Curriculum policies for ELs in mathematics should follow the guidelines for traditionally underserved students (AERA, 2006), such as instituting systems that broaden course-taking options and avoiding systems of tracking students that limit their opportunities to learn and delay their exposure to college-preparatory mathematics coursework.

ⁱⁱⁱ For examples of lessons where ELs participate in mathematical discussions, see Moschkovich, 1999 and Khisty, 1995.

^{iv} Topics for further research include defining linguistic complexity for mathematical texts and providing examples of linguistic complexity that go beyond readability (such as the syntactic structure of sentences, underlying semantic structures, or frequency of technical vocabulary, verb phrases, conditional clauses, relative clauses, and so on).

^v For examples of how assessment and instruction can focus on mathematical content and reasoning see Appendix A, Moschkovich (1999) and Moschkovich (2007a).

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Appendix A: A Classroom Vignette

This vignette is presented to ground the subsequent descriptions of the recommendations and to show how these recommendations play out in classroom interactions. The lesson excerpt presented below (Moschkovich, 1999) comes from a third-grade bilingual classroom in an urban California school¹. In this classroom, there were thirty-three students identified as Limited English Proficient. In general, this teacher introduced students to topics in Spanish and then later conducted lessons in English. The students had been working on a unit on two-dimensional geometric figures. For several weeks, instruction had included vocabulary such as “radius,” “diameter,” “congruent,” “hypotenuse,” and the names of different quadrilaterals in both Spanish and English. Students had been talking about shapes and the teacher had asked them to point, touch, and identify different shapes. The teacher identified this lesson as an English as a Second Language mathematics lesson, one where students would be using English in the context of folding and cutting to make Tangram pieces (see Figure 1).

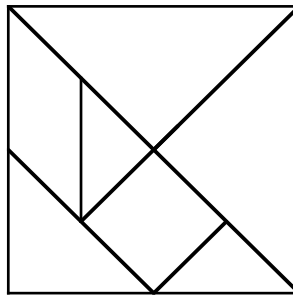


Figure 1: A tangram puzzle

Vignette

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|----|----------|---|
| 1. | Teacher: | Today we are going to have a very special lesson in which you really gonna have to listen. You're going to put on your best, best listening ears because I'm only going to speak in English. Nothing else. Only English. Let's see how much we remembered from Monday. Hold up your rectangles . . . high as you can. (Students hold up rectangles) Good, now. Who can describe a rectangle? Eric, can you describe it [a rectangle]? Can you tell me about it? |
| 2. | Eric: | A rectangle has . . . two . . . short sides, and two . . . long sides. |
| 3. | Teacher: | Two short sides and two long sides. Can somebody tell me something else about this rectangle, if somebody didn't know what it looked like, what, what . . . how would you say it. |
| 4. | Julian: | Paralela [holding up a rectangle, voice trails off]. |
| 5. | Teacher: | It's parallel. Very interesting word. Parallel. Wow! Pretty interesting word, isn't it? Parallel. Can you describe what that is? |

6.	Julian:	Never get together. They never get together [runs his finger over the top side of the rectangle].
7.	Teacher:	What never gets together?
8.	Julian:	The paralela . . . they . . . when they go, they go higher [runs two fingers parallel to each other first along the top and base of the rectangle and then continues along those lines], they never get together.
9.	Antonio:	Yeah!
10.	Teacher:	Very interesting. The rectangle then has sides that will never meet. Those sides will be parallel. Good work. Excellent work.

The vignette serves to show that English language learners can and do participate in discussions where they grapple with important mathematical content. Students were grappling not only with the definitions for quadrilaterals but also with the concept of parallelism. Student were engaged in mathematical communication because they were making claims, generalizing, imagining, hypothesizing, and predicting what will happen to two lines segments if they are extended indefinitely. To communicate about these mathematical ideas students used words, objects, gestures, and other students' utterances as resources. This vignette also illustrates several instructional strategies that can be useful in supporting student participation in mathematical discussions. Some of these strategies are: asking for clarification, re-phrasing student statements, accepting and building on what students say, and probing what students mean. It is important to notice that this teacher did *not* focus directly on vocabulary development but instead on mathematical ideas and arguments as he interpreted, clarified, and rephrased what students were saying. This teacher provided opportunities for discussion by moving past student grammatical or vocabulary errors, listening to students, and trying to understand the mathematics in what students said. He kept the discussion mathematical by focusing on the mathematical content of what students said and did.

Recommendation #1: Focus on Students' Mathematical Reasoning, Not Accuracy in Using Language.

In the vignette: Uncovering the mathematical content in Julian's contributions is certainly a complex endeavor. Julian's utterances in turns 4, 6, and 8 are difficult both to hear and interpret. He uttered the word "paralela" in a halting manner, sounding unsure of the choice of word or of its pronunciation. His voice trailed off, so it is difficult to tell whether he said "paralelo" or "paralela." His pronunciation could be interpreted as a mixture of English and Spanish; the "ll" sound being pronounced in English and the addition of the "o" or "a" being pronounced in Spanish. The grammatical structure of the utterance in line 8 is intriguing. The apparently singular "paralela" is preceded by the word "the" which can be either plural or singular and then followed with a plural "when they go higher." In any case, what is clear is that Julian made several attempts to communicate a mathematical idea in his second language. If we only focus only on his English proficiency, we would miss his mathematical reasoning. Julian is, in fact, accurately describing a property of parallel lines. This teacher moved past Julian's unclear utterance and use of the term "paralela." He focused on the mathematical content of what students said, not the mistakes they made. He attempted to uncover the mathematical content in what Julian had said. He did not correct Julian's English, but instead asked questions to probe what the student meant.

Recommendation #2: Shift to a Focus on Mathematical Discourse Practices, Move Away from Simplified Views of Language.

In the vignette: What competencies in mathematical practices did Julian display? Julian was participating in three central mathematical practices: abstracting, generalizing, and imagining. He was describing an *abstract* property of parallel lines and making a generalization saying that parallel lines will *never* meet. He was also imagining what happens when the parallel sides of a rectangle are extended. If we only focused on vocabulary, we would miss Julian's use of these important mathematical practices.

Recommendation #3: Recognize and Support Students to Engage with the Complexity of Language in Mathematics Classrooms.

In the vignette: What modes of expression did Julian and the teacher use? Julian used gestures and objects in his description, running his fingers along the parallel sides of a paper rectangle. The teacher also used gestures and visual displays of geometric figures on the blackboard. This example shows some of the complexity of language in the mathematics classroom.

Recommendation #4: Treat Everyday Language and Experiences as Resources, Not as Obstacles.

In the vignette: What language resources did Julian use to communicate his mathematical ideas? He used colloquial expressions such as "go higher" and "get together" rather than the formal terms "extended" or "meet." These everyday expressions were not obstacles but resourcesⁱⁱ.

Recommendation #5: Uncover the Mathematics in What Students Say and Do.

In the vignette: How did the teacher respond to Julian's contributions? The teacher moved past Julian's confusing uses of the word "paralela" to focus on the mathematical content of Julian's contribution. He did not correct Julian's English, but instead asked questions to probe what the student meant. This response is significant in that it represents a stance towards student contributions during mathematical discussion: listen to students and try to figure out what they are saying. When teaching English learners, this means moving beyond vocabulary, pronunciation, or grammatical errors to listen for the mathematical content in student contributions. (For a discussion of the tensions between these two, see Adler, 2001.)

In the vignette: What instructional strategies did the teacher use? The teacher used gestures and objects, such as the cardboard geometric shapes, to clarify what he meant. For example, he pointed to vertices and sides when speaking about these parts of a figure. Although using objects to clarify meanings is an important ESL instructional strategy, it is crucial to understand that these objects do not have meaning that is *separate* from language. Objects acquire meaning as students talk about them and these meanings are negotiated through talk. Although the teacher and the students had the geometric figures in front of them, and it seemed helpful to use the objects and gestures for clarification, students still needed to sort out what 'parallelogram' and 'parallel' meant by using language and negotiating common meanings for these words.

In the vignette: The teacher did not focus on vocabulary instruction but instead supported students' participation in mathematical arguments by using three instructional strategies that focus more on mathematical discourse: 1) **Building on student responses:** The teacher accepted and built on student responses. For example in turns 4-5, the teacher accepted Julian's response and probed what he meant by "parallel." 2) **Asking for clarification:** The teacher prompted the students for clarification. For example, in turn 7 the teacher asked Julian to clarify what he meant by "they." 3) **Re-phrasing:** The teacher re-phrased (or re-voiced) student statements, by interpreting and rephrasing what students said. For example, in turn 10 the teacher rephrased what Julian had said in turn 8. Julian's "the parallela, they" became the teacher's "sides" and Julian's "they never get together" became "will never meet". The teacher thus built on Julian's everyday language as he re-voiced Julian's contributions using more academic language.

ⁱ This work was supported by Grants #REC-9896129 and #ROLE-0096065 from NSF. The Math Discourse Project at Arizona State University videotaped this lesson with support by an NSF grant.

ⁱⁱ The question of whether mathematical ideas are as clear when expressed in colloquial terms as when expressed in more formal language is highly contested and not yet, by any means, settled. For a discussion of this issue, see Tim Rowland's book *The Pragmatics of Mathematics Education: Vagueness in Mathematical Discourse*.